Dispersion reducing finite difference coefficients for marine controlled source electromagnetic application

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Summary

A combination of the FDTD technique and Mittet's mapping method is a powerful and fast numerical tool for solution of marine controlled source electromagnetic problems. The efficiency of this technique can be further improved by using optimized, dispersion reducing finite difference coefficients for calculation of first derivatives in Maxwell's equations. We present a method for calculating such coefficients. It is demonstrated that by utilizing dispersion reducing coefficients, the discretization mesh size can be reduced up to two times in 3D diffusive problems.

Introduction

The marine controlled source electromagnetic (CSEM) method is a low-frequency electromagnetic imaging technique aimed at generating the electric resistivity map of seafloors. Due to its high relative cost, the method has only recently been employed on a commercial scale as a tool for hydrocarbon exploration (A. H. Bhuiyan, 2009, S. Constable and L. J. Srnka, 2007). However, with the improvement in speed of the computational devices and effectiveness of the algorithm, CSEM acceptance in the hydrocarbon exploration industry is steadily growing.

In a CSEM survey, an electric dipole transmitter and arrays of electromagnetic (EM) receivers are used to probe the subsurface. The recorded data are used in an EM inversion process to obtain the required resistivity map. Due to the high electrical loss of the medium through which the EM wave propagates, the operational frequencies used in marine CSEM are low, typically between 0.1-5 Hz. Therefore, CSEM is a low resolution technique not suitable for accurate reconstruction of the exact geometry of the seafloor subsurface layers. For this reason, CSEM is mostly used as a complementary tool to high-resolution acoustic-based seismic imaging techniques, which can reconstruct the subsurface stratification quite precisely, but cannot confirm the existence of hydrocarbons. CSEM technique, on the other hand, has the ability to detect the presence of hydrocarbons with a high confidence. Therefore, this technique is used to differentiate between reservoirs that contain hydrocarbons and those that are of no commercial interest.

An EM inversion requires a very large number of forward EM simulations at multiple frequencies. Therefore, the speed of each EM simulation is critical to lowering the cost of the inversion. Currently, most of the industry uses frequency-domain EM techniques such as the frequency-domain finite difference (FDFD) method (G.A.Newman and D.L.Alumbaugh, 1995). These methods are robust; however they require various simplifying assumptions to reduce the memory and computational burden, e.g.: uniform or layered background medium. The finite difference time domain (FDTD) technique is an alternative to frequency domain techniques. Some of the advantages of the FDTD include: arbitrarily shaped and anisotropic scatterer and background medium, ease of parallelization, broad frequency response in a single run. However, the main disadvantage of the FDTD is a very small timestep required by the stability criterion, leading to a prohibitively large number of required iterations. Several techniques have been suggested in the literature to alleviate this problem, such as artificially increasing electric permittivity of the medium (T. Wang and G.W.Hohmann, 1993, Maao 2007).

Recently, Mittet has developed a technique which allows significant reduction of the number of time iterations in the FDTD simulations of diffusive EM problems (R. Mittet, 2010). In this paper, we show that the efficiency these simulations can be further improved by using dispersion reducing finite difference (FD) coefficients for calculation of the first derivatives used in Maxwell's equations. We demonstrate that by using the dispersion reducing coefficients (DRC) instead of traditional (*Taylor*) coefficients, the discretization mesh size can be reduced by a factor of two, significantly improving the overall speed of the calculations.

Fictitious Domain

CSEM problems are diffusive problems where the EM wave propagates through a highly lossy medium. The stability limit used in the conventional FDTD algorithm dictates a very small timestep, leading to a prohibitively large number of iterations in a single run. A promising approach to reducing the cost of the FDTD solution of diffusive problems is proposed by Mittet (R. Mittet, 2010). The time-domain simulation is carried out in a fictitious domain where material parameters are obtained from the parameters in the diffusive domain according to

$$\boldsymbol{\epsilon}'(\boldsymbol{x}) = \frac{\boldsymbol{\sigma}(\boldsymbol{x})}{2\omega_0} \qquad (1)$$

where ω_0 is an arbitrary scaling radial frequency. Following Mittet's notation, bold characters represent tensors, while primed characters represent values specific to the fictitious domain. The materials in the fictitious domain are lossless with very high values of relative dielectric constant. Therefore, the velocity of the EM wave in the fictitious domain is low, allowing use of a relatively large timestep in the FDTD simulations.

Using (1), time-domain quasi-static Maxwell's equations in the diffusive domain are transformed to a set of time-domain hyperbolic partial differential equations describing the EM field in the fictitious domain. The latter are given by

$$-\nabla \times \boldsymbol{H}' + \boldsymbol{\epsilon}' \frac{\partial \boldsymbol{E}'}{\partial t'} = -\boldsymbol{J}'_{src}$$
$$\nabla \times \boldsymbol{E}' + \boldsymbol{\mu} \frac{\partial \boldsymbol{H}'}{\partial t'} = -\boldsymbol{M}'_{src} \quad (2)$$

where $\boldsymbol{J}'_{src}$ and $\boldsymbol{M}'_{src}$ are electric and magnetic current densities, respectively. The frequency-domain EM fields in the diffusive and fictitious domains are related by

$$\boldsymbol{E}'(\boldsymbol{x}, \omega') = \boldsymbol{E}(\boldsymbol{x}, \omega)$$
$$\boldsymbol{H}'(\boldsymbol{x}, \omega') = \sqrt{\frac{-j\omega}{2\omega_0}} \boldsymbol{H}(\boldsymbol{x}, \omega)$$
$$\boldsymbol{J}'(\boldsymbol{x}, \omega') = \sqrt{\frac{-j\omega}{2\omega_0}} \boldsymbol{J}(\boldsymbol{x}, \omega) \quad (3)$$
$$\boldsymbol{K}'(\boldsymbol{x}, \omega') = \boldsymbol{K}(\boldsymbol{x}, \omega)$$

The radial frequency in the fictitious domain can be expressed as

$$\omega' = (j+1)\sqrt{\omega\omega_0} \quad (4)$$

Partial differential equations (2) have the same form as regular time-domain Maxwell's equations. Therefore, they can be solved in the fictitious domain by a standard FDTD algorithm. The obtained fields contain enough information to recover the fields in the diffusive domain. The frequency-domain fields in the diffusive domain can be obtained directly from the time-domain fields in the fictitious domain by using a "Modified" Fourier Transform (MFT) given by

$$E_i(x, \omega) = \int_0^T E_i'(x, t') e^{-\sqrt{\omega\omega_0}t'} e^{j\sqrt{\omega\omega_0}t'} dt'$$
$$H_i(x, \omega) = \sqrt{\frac{-2\omega_0}{j\omega}} \int_0^T H_i'(x, t') e^{-\sqrt{\omega\omega_0}t'} e^{j\sqrt{\omega\omega_0}t'} dt' \quad (5)$$
$$i \in \{x, y, z\}$$

It is important to notice that MFT (5) contains an exponential attenuating term $e^{-\sqrt{\omega\omega_0}t'}$, which damps all late time arrivals. Using this method, the number of timesteps in a typical simulation can be reduced by several orders of magnitude, while preserving the high accuracy of a full wave technique.

Dispersion-Reducing FD Coefficients

In a broad range of RF and photonic problems, $\lambda/10$ or smaller discretization cells are required by the geometry of the structure under analysis. In these cases, the second order in space FDTD technique is adequate for most of the simulations. However, in CSEM problems utilizing Mittet's technique, dimensions of the geological structures are relatively large comparing to the wavelength. In this type of problems, the cell size is dispersion-limited; therefore, a higher order FD approximation of the first derivative is beneficial because it allows larger cells and smaller mesh sizes. The cell size can be increased even further by optimizing the FD coefficients for minimum spatial dispersion. In this respect, the approach outlined in (R. McGarry et al., 2011) can be applied to FDTD method as well.

The $2N$-point FD approximation of the first derivative of a function $f(x)$ defined on a uniform staggered grid can be expressed as

$$\frac{\partial f(x)}{\partial x} \approx \frac{1}{\Delta \mathrm{x}} \sum_{n=1}^{N} a_n [f(x + r_n \Delta \mathrm{x}) - f(x - r_n \Delta \mathrm{x})] \quad (6)$$

where Δx is the spatial step, $r_n = (2n-1)/2$ and a_n denotes the FD coefficients. In wavenumber domain, (6) can be expressed as

$$K \approx \sum_{n=1}^{N} 2a_n \sin(r_n K) \quad (7)$$

where $K = k\Delta$x, and k is the wavenumber. The FD coefficients a_n can be optimized to maximize the range of K over which (7) is sufficiently accurate for any given number N. This is done by using a least square (LS) optimization with an error function E defined by

$$E = \int_0^{K_m} \left[\sum_{n=1}^{N} 2a_n \sin(r_n K) - K \right]^2 dK \quad (8)$$

K_m defines the range over which the coefficients are optimized. To minimize the error function, the following set of equations needs to be solved

$$\frac{\partial E}{\partial a_p} = 0, \;\; p = 1,2,\ldots,N \qquad (9)$$

Solving system (9) leads to a set of N LS-optimized FD coefficients which decrease spatial dispersion for coarse cells. However, for very small spatial steps, unnecessarily large dispersion error is obtained. This problem can be avoided by combining system (9) with a set of standard or *Taylor* equations given by

$$\sum_{n=1}^{N} a_n r_n^q = \begin{cases} 1/2 & q = 1 \\ 0 & q = 1,3,\ldots,2N-1 \end{cases} \qquad (10)$$

The above equations are obtained by expanding the sine function on the right-hand side of (7) about $K = 0$ using Taylor series and equating the first N coefficients of the resulting series with the left-hand side of (7). Numerical tests have shown that the best performance of the FD coefficients is obtained by combining the first two Taylor equations ($q = 1,3$) with $N-2$ LS equations ($p = 1,2,\ldots,N-2$). This particular combination of the equations is used to calculate the optimized FD coefficients which we denote as dispersion reducing coefficients (DRC).

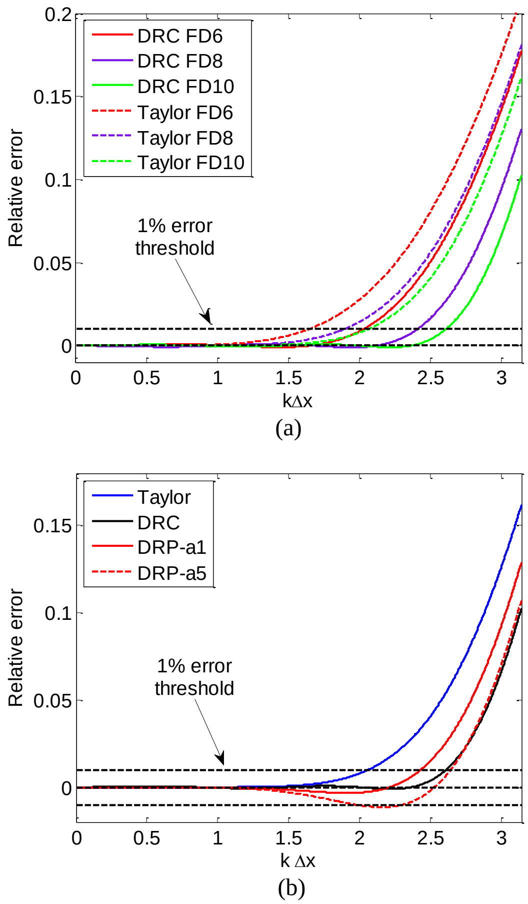

Figure 1 Relative spatial dispersion error obtained with (a) DRC and Taylor coefficients and FD 6-10, (b) Taylor, DRC and DRP coefficients and FD 10. DRP-a1 and DRP-a5 denote DRP coefficients obtained using the LS equation with p=1 and p=5, respectively.

To demonstrate the effectiveness of DRC, the relative error of approximation (7) is illustrated in Figure 1 for different FD stencil lengths. Clearly, DRC significantly increase the range of the normalized wavenumbers K over which the relative error is below a certain threshold. For example, if 1% error threshold is allowed, the 8-point FD scheme requires a normalized cell size of 1.89 or 2.41, corresponding to 3.3 or 2.6 cells per wavelength for Taylor or DRC coefficients, respectively. This means that DRC allow approximately 27% larger cell size over Taylor coefficients without loss of accuracy. For 3D CSEM problems, this translates into a factor of 2 reduction of the overall mesh size.

For completeness, the performance of DRC is compared with another type of optimized coefficients, dispersion-relation-preserving (DRP) coefficients (Ye & Chu, 2005). The later are obtained by solving first $N-1$ Taylor equations together with a single LS equation. Figure 1(b) compares the dispersion errors obtained with 10-point FD scheme with DRC and DRP coefficients obtained using the LS equation with $p = 1$ and $p = 5$ (corresponding to the first and last coefficient). Clearly, DRC exhibit the best performance. The error obtained using DRP coefficients shows unacceptably large ripples close to the upper end of the passband.

Numerical Example

The effectiveness of DRC is demonstrated on a numerical example involving wave propagation in a 3D diffusive uniform medium, set as seawater with a resistivity of $0.3\,\Omega$m. Using (1) and a scaling frequency of $1\,Hz$, this diffusive medium maps into a lossless fictitious medium with a dielectric constant of $0.2653\,F/m$. The system is excited at the center of the computational domain with an x-direction oriented electrical dipole emitting a transient signal which in the fictitious domain behaves as the fist derivative of a Gaussian

$$s(t') = -2\beta(t'-t_0)\sqrt{\beta/\pi}\,e^{-\beta(t'-t_0)^2} \qquad (11)$$

The above equation parameters are defined as $\boldsymbol{\beta = \pi f_{max}^2}$ and $\boldsymbol{t_0 = \pi / f_{max}}$. The maximum frequency is chosen to be $\boldsymbol{f_{max} = 3\,Hz}$.

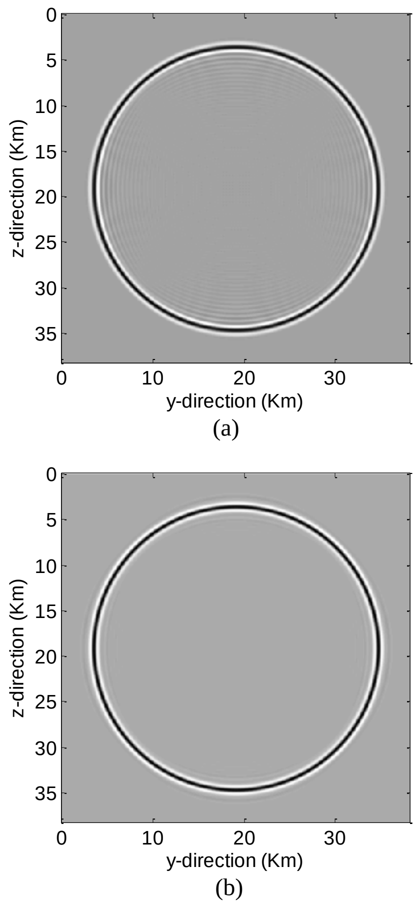

Figure 2 Forward propagating Ex field component in the fictitious domain after 11 seconds. The fields are obtained using the 10-point in space FDTD technique with (a) Taylor coefficients, (b) DRC.

The simulation is performed using the FDTD method with 10-point FD scheme. The discretization cells are cubic with an edge size of $\mathbf{180\ m}$. The $\boldsymbol{E_x}$-field on a y-z plane through the source location after 11 seconds, obtained using both Taylor and DRC coefficients is shown in Figure 2(a) and (b), respectively. The ripples trailing the main wave front due to the numerical dispersion are clearly visible when Taylor coefficients are used, while DRC produce no such ripples.

Finally, the $\boldsymbol{E_x}$ field component is sampled 1.8 Km from the source location in y-direction. The signals obtained with Taylor coefficients and DRC are compared with a reference signal obtained using a very fine discretization with 25-m cell size. The comparison is presented in Figure 3. Strong spatial dispersion effects are clearly visible with the Taylor coefficients, while our DRC coefficients lead to excellent agreement with the reference result, with very small dispersive ripples just starting to appear at the tail end.

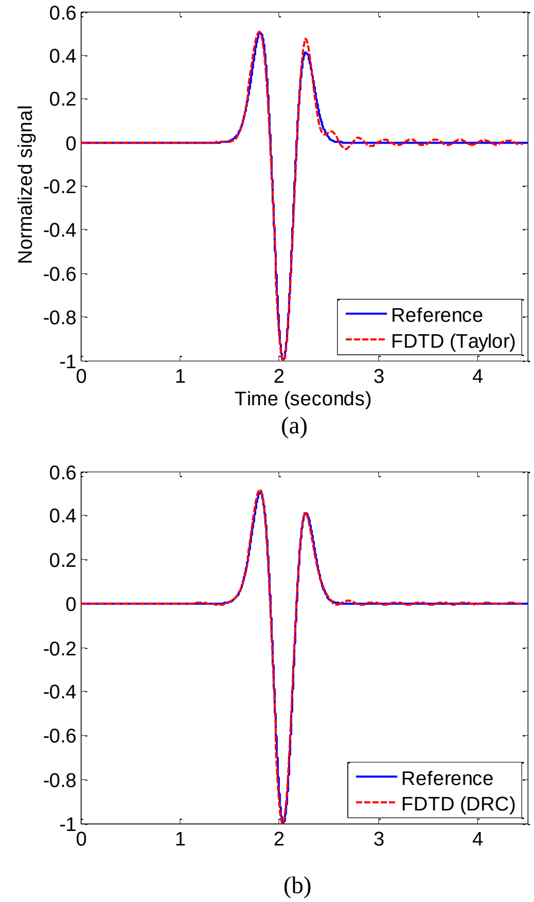

Figure 3 Transient Ex field observed 1.8 Km from the source location. The field is obtained using 10-point in space FDTD technique with (a) Taylor coefficients, (b) DRC.

Conclusions

The FDTD technique combined with Mittet's method is a powerful numerical tool which can be used in marine CSEM for survey designs and data inversion. Since it is a time-domain technique, multiple frequency results can be obtained from a single run. Utilizing Mittet's mapping method allows larger simulation timestep, greatly improving the speed of the simulation. We have shown that the efficiency of the method can be further improved by using a higher order FD scheme with optimized coefficients for calculation of spatial derivatives. The effectiveness of our DRC formulation has been demonstrated on an example of a uniform 3D lossy medium.