

## Introduction

In the exploration of areas with complex salt bodies, such as the Gulf of Mexico, RTM is often run iteratively for testing different scenarios of the base of salt and scanning different sub-salt velocity models. However, RTM is still an expensive tool, especially for large 3D Tilted Transverse Isotropic (TTI) media. To speed up the iteration of RTM when the upper part of the velocity model is kept invariant, Wang et al., (2011), Ji et al., (2012) and Guan et al., (2008) proposed a layer-stripping RTM. The reduction of computation cost and memory requirement is due to a number of factors, including: 1) the wavefields above the redatum depth are propagated only once; 2) computation grid size and time step can be greatly increased because the minimum velocity typically increases with depth; 3) model size, wavefield propagation time and migration aperture are also reduced when migrating from a subsurface redatum depth; 4) the dip field may be less sensitive in the deeper regions, so that VTI or isotropic RTM may be sufficient, even with a TTI overburden; 5) For each region, the RTM memory requirement is much less, making it possible to run RTM on hardware configurations with less available memory.

In the implementation of layer-stripping RTM, the first key issue is the data explosion problem for storing 3D redatumed source and receiver wavefields at every computation grid on the redatum depth. The second key issue is the amplitude-preserved injection of the redatumed source and receiver wavefields using time-domain finite-difference method. To solve the first issue, Ji et al., (2011) employed a wavelet compression algorithm to achieve compression ratios of up to two orders of magnitude without significantly damaging the quality of the redatumed data. But the second key issue has not yet been explicitly addressed. In this paper, we will show that with improper injection of the redatumed wavefields, the relative amplitude of individual source and receiver wavefields will be distorted, compared to regular source and receiver wavefields without using layer stripping. Then, we present a new formulation for injecting redatumed source and receiver wavefields into the acoustic model for finite-difference method, while preserving their relative amplitude. In this new formulation, a directional correction factor and the effect of velocity and grid size at the injection location are incorporated. We also demonstrate its effectiveness by comparing true-amplitude layer-stripping acoustic RTM and regular true-amplitude acoustic RTM with synthetic and field datasets.

## Theory

As described by Wang et al., (2011), for layer-stripping RTM, the model is first divided into two or three horizontal regions, in which RTM is run sequentially in a top-down approach. For the top region, a regular RTM is run and the source and receiver wavefields at the bottom of the region are saved. These saved redatumed wavefields become the input for the subsequent RTM to image the region below. The wavefield redatuming simply means saving the source and receiver wavefields at the bottom of top region, i.e., the redatum depth. Thus, a point source on the surface becomes an area source on the subsurface datum. So layer-stripping RTM needs at least 1 layer overlap between the upper region and adjacent lower region.

In this study, we first use the true-amplitude common-shot RTM (Qin & McGarry, 2013) to image the top region by using the regular shot gather recorded at the surface. During this step, we record the redatumed source and receiver wavefields. Then, the redatumed source wavefield is forward propagated using a standard finite-difference method as

$$P_D(\mathbf{x}; \omega) = \int S'(\mathbf{x}_d; \omega) G(\mathbf{x}_d, \mathbf{x}; \omega) \frac{\cos(\alpha_d^s) v_d^s}{\Delta x \Delta y \Delta z} d^2 \mathbf{x}_d \quad (1)$$

where  $P_D(\mathbf{x}; \omega)$  is the propagated source wavefield below the redatum depth;  $S'(\mathbf{x}_d; \omega)$  is the fully-differentiated redatumed source wavefield, where full differentiation is applied to counteract Huygens summation.  $G(\mathbf{x}_d, \mathbf{x}; \omega)$  is the Green function between redatumed source wavefield  $\mathbf{x}_d$  and image point  $\mathbf{x}$ ;  $v_d^s$  is the wave speed at the injection point of redatumed source wavefield.  $\alpha_d^s$  is the emergent angle, i.e., the angle between the ray from  $\mathbf{x}_d$  to  $\mathbf{x}$  and the downward normal to the redatum surface.

$\Delta x$ ,  $\Delta y$  and  $\Delta z$  are the grid size at redatum depth. The redatumed receiver is backward propagated similarly as

$$P_U(\mathbf{x}; \omega) = \int R'(\mathbf{x}_d; \omega) G^*(\mathbf{x}_d, \mathbf{x}; \omega) \frac{\cos(\alpha_d^r) v_d^r}{\Delta x \Delta y \Delta z} d^2 \mathbf{x}_d \quad (2)$$

where  $P_U(\mathbf{x}; \omega)$  is the backward-propagated receiver wavefield below the redatum depth;  $R'(\mathbf{x}_d; \omega)$  is the fully-differentiated redatumed receiver data.  $G^*(\mathbf{x}_d, \mathbf{x}; \omega)$  is the complex conjugate of the Green function between redatumed receiver wavefield  $\mathbf{x}_d$  and image point  $\mathbf{x}$ .  $v_d^r$  is the wave speed at the injection point of redatumed receiver wavefield.  $\alpha_d^r$  is the emergent angle, i.e., the angle between the ray from  $\mathbf{x}_d$  to  $\mathbf{x}$  and the downward normal to the redatum surface.  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  are the grid size at redatum depth.

To illustrate the necessity of the directional terms in equations (1) and (2), we compare snapshots of the source wavefield after injecting the redatumed data at a depth of 2500m in a constant velocity, with and without this directional correction. As seen in Fig. 1a and 1c, the amplitude is spherically symmetric as expected when this directional term is applied. Without this directional term, Fig. 1b and 1d show that the amplitude increases with increasing emergent angle.

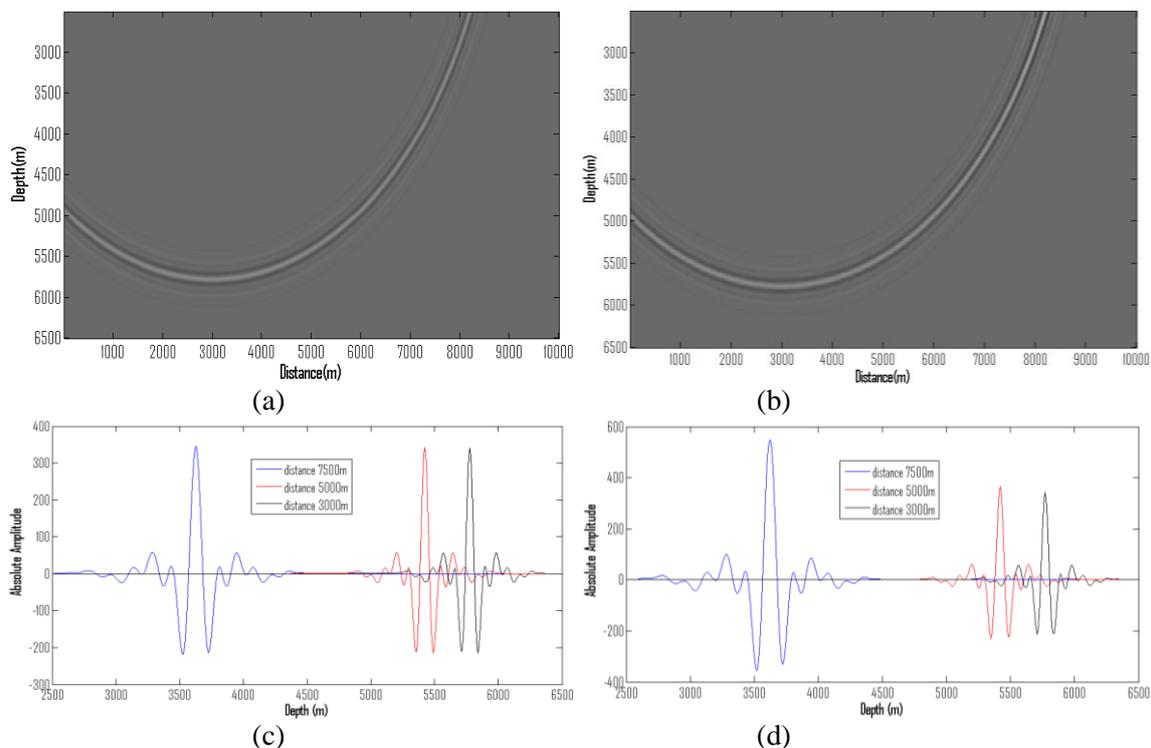
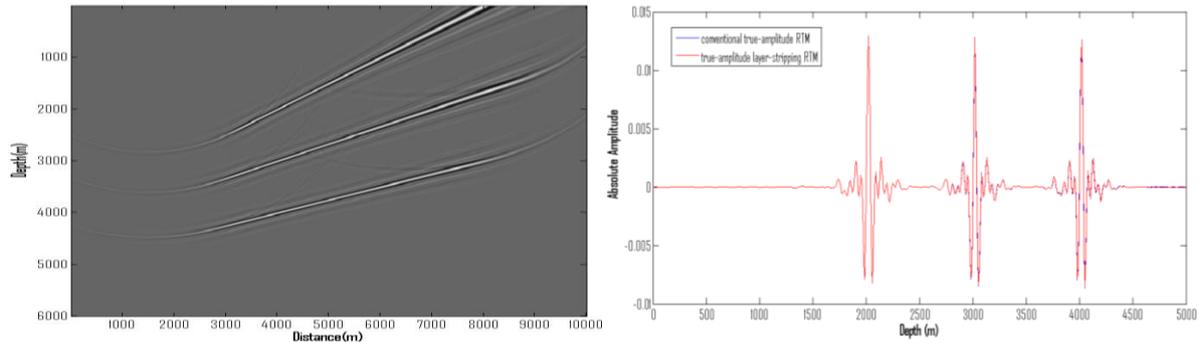


Figure 1: Snapshot of source wavefield after injecting the redatumed data at a depth of 2500m with directional correction (a) and without directional correction (b). (c) and (d) compare traces in (a) and (b), respectively.

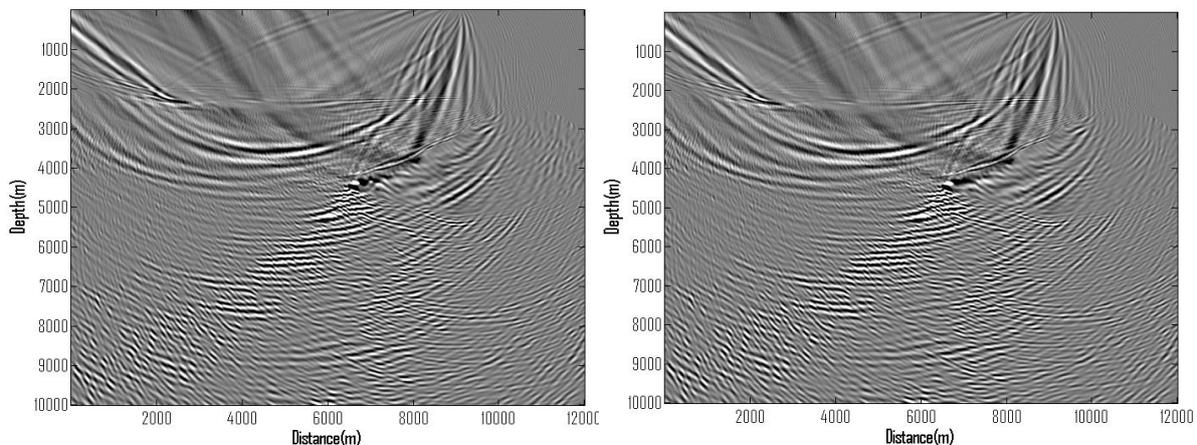
## Numerical and Field Examples

To show that our true-amplitude layer-stripping RTM produces true-amplitude shot images, comparable to those of regular true-amplitude common-shot RTM, we first tested our method using a 2D synthetic model with 3 dipping density-contrast reflectors and constant velocity of 3000m/s. These 3 reflectors have the same theoretical angle-independent reflectivity. The input synthetic shot gather is generated without including source and receiver ghosts. For testing layer-stripping RTM, the model is divided along a depth of 2500m as indicated by the dotted green line in the left of Fig. 2. The regular true-amplitude common-shot RTM (Qin & McGarry, 2013) is used to image the upper part and generate the redatumed source and receiver wavefields at the depth of 2500m.

The left of Fig. 2 shows the true-amplitude layer-stripping RTM shot image. It can be seen that although there is only 1 cell overlap between upper layer and lower layer, the footprint caused by model division at depth of 2500m is not visible on the image. The right of Fig. 2 shows the comparison of the image trace at a distance of 4000m, from which it can be seen that: 1) the imaged reflectors with different depth and dip have the same amplitude, as predicted by theory; 2) the true-amplitude layer-stripping RTM gives almost identical result as regular true-amplitude RTM.



**Figure 2:** Left: true-amplitude layer-stripping RTM shot image where the redatum depth is 2500m. Right: comparison of the image trace at distance of 4000m between regular true-amplitude RTM and true-amplitude layer-stripping RTM. The model has 3 dipping density-contrast reflectors and constant background velocity of  $v=3000\text{m/s}$ .



**Figure 3:** Comparison of regular true-amplitude RTM (left) and true-amplitude layer-stripping RTM (right) where the redatum depth is at 4500m and cuts across the salt body.

The second example is a complex 2D field model with a salt body, in a deep-water region of Gulf of Mexico. Figure 3 compares the raw shot image using regular true-amplitude RTM and the shot image using the true-amplitude layer-stripping RTM, where the model division cuts across the salt body as indicated by the dotted green line. These 2 images look identical. A quantitative comparison of the image traces at a distance of 8000m is shown in Fig. 4a, where the small difference above the redatum depth can be attributed to the salt-body interbed multiples which are not accounted for since the redatum depth divides the salt body. By contrast, Fig. 4b shows the similar comparison, but the redatum depth is now below the salt body. Now the image above the redatum depth is identical except for the area close to the redatum depth which appears to be caused by artificial absorbing boundary reflection. From Fig. 4, it can also be noted that the image below the redatum depth is almost identical between our true-amplitude layer-stripping RTM and regular true-amplitude RTM. This illustrates the effectiveness of the method for injecting redatummed source and receiver wavefields.

## Conclusions

In this paper, we first presented a new formulation for injecting redatumed wavefields into the acoustic model for finite-difference method while preserving the relative amplitude below the redatum depth. Our approach needs only one layer overlap between upper and lower regions. Then we developed a true-amplitude layer-stripping common-shot acoustic RTM by combining regular true-amplitude acoustic RTM with the amplitude-preserved injection of redatumed source and receiver wavefields. The true-amplitude layer-stripping common-shot acoustic RTM can give nearly identical results to regular true-amplitude acoustic RTM if the velocity model is properly divided.

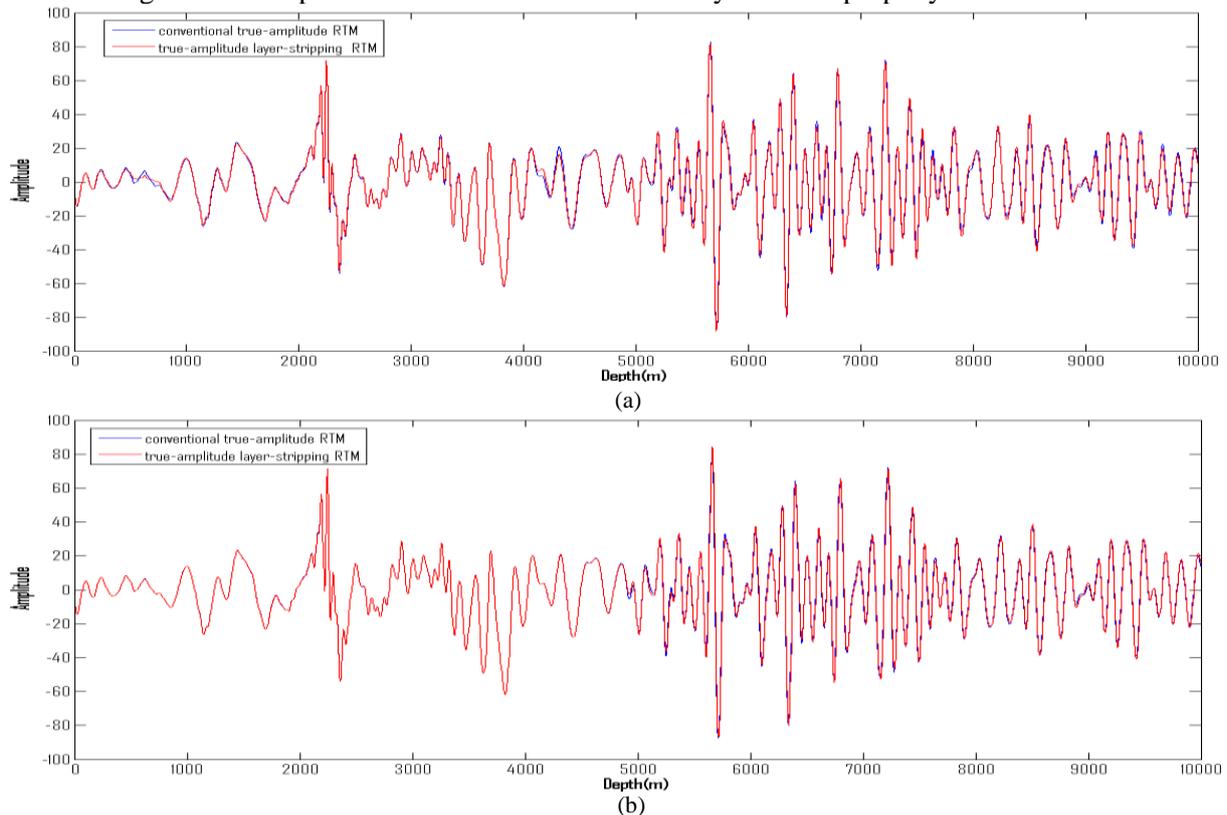


Figure 4: Comparison of the image trace at distance=8000m between regular true-amplitude RTM and true-amplitude layer-stripping RTM for different redatum depths. (a) redatum depth is 4500m and cuts across the salt body. (b) the redatum depth is 5250m and below the high-velocity salt body.

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## References

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