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Temporal Dispersion Correction and Prediction by Using Spectral Mapping

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Summary

Time-domain finite difference (FD) modeling for wave propagation has been widely used for illumination studies and advanced imaging techniques such as RTM and FWI in complex geology. Any finite-order approximation of the time and space derivatives using FD methods suffer from some degree of numerical dispersion. The temporal and spatial dispersion are independent of each other. Spatial dispersion can be reduced with higher-order finite difference operators and optimized coefficients. The time derivatives are usually approximated to 2nd order. Thus, FD-based simulations and imaging methods often suffer from numerical temporal dispersion error.

In this paper, we show that the numerical temporal dispersion from 2nd-order time FD can be corrected via a trace-by-trace spectral mapping operation. For RTM, this spectral mapping operation is to add the predicted temporal dispersion into each gather before backward propagation. This approach eliminates the temporal dispersion at only small cost. One clear benefit of the temporal dispersion correction is a significant speed up because a larger time step can be used.

Introduction

As the demand for high-resolution reservoir model in complex geological structure increases, the research of seismic imaging and inversion focuses on 2-way wave-equation based methods such as RTM and FWI. The accuracy and efficiency of broadband wavefield modeling is key to the success of these methods. The 2-way wave-equation is often solved by using time-domain finite-difference (FD) methods. A finite difference solution applies approximate derivative operators to discretize the time and space derivatives. The resultant numerical dispersion due to approximate derivative operators can introduce severe numerical error in the synthetic data, migration image and FWI gradient.

Numerous approaches have been developed to reduce the space dispersion, such as the higher-order FD scheme, optimized FD operators (McGarry et al., 2011) or Fourier methods. Unlike the spatial derivatives, temporal derivatives are usually approximated in 2nd order to reduce memory requirements and computation costs. As a result, the FD-based simulation and imaging method often suffers from temporal dispersion error. The time dispersion only depends on the frequency, time step size, and propagation time, not velocity model. Thus, time dispersion is predictable and could be handled separately from spatial dispersion. Stork (2013) proposes to reduce the time dispersion by applying a time variant filter and interpolation. Following Stork's approach, Li et al. (2016) showed two post-propagation filtering schemes. Dai et al. (2014) show more details and mathematical analysis of the temporal dispersion error from the 2nd-order FD of the time derivatives and proposed a method of time varying phase shift (TVPS). Wang & Xu (2015) proposed a modified Fourier transform to correct the phase error from the temporal dispersion. These types of temporal dispersion corrections do not add significant computational costs and can be applied to each seismic trace before back-propagation in RTM or after simulation for FD modeling.

In this paper, we show that the numerical temporal dispersion from 2nd-order time FD can be corrected or predicted via a trace-by-trace spectral mapping operation. This method is accurate and efficient. To compensate the temporal dispersion in RTM imaging, the predicted dispersion is added into input traces before backward propagation. We demonstrate the accuracy and effectiveness of the method with a synthetic modeling example and a real RTM example in complex media.

Theory

The acoustic wave equation can be written as

$$\frac{\partial^2 u(x,t)}{\partial t^2} = -v^2(\boldsymbol{x})\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}\right)u(\boldsymbol{x},t) \quad (1)$$

Assuming a constant velocity, in the time-wavenumber domain, the acoustic wave equation become

$$\frac{d^2 u(\boldsymbol{k},t)}{dt^2} = -v^2 k^2 u(\boldsymbol{k},t) \quad (2)$$

where $k = |\boldsymbol{k}|$. Using 2nd-order temporal FD, the solution of equation (2) is

$$u(\boldsymbol{k},t+\Delta t) = (2 - v^2\Delta t^2 k^2)u(\boldsymbol{k},t) - u(\boldsymbol{k},t-\Delta t) \quad (3)$$

To analyze the time dispersion, we apply Fourier transform to transform equation (3) into frequency-wavenumber domain.

$$u(\boldsymbol{k},\omega)e^{i\omega\Delta t} = (2 - v^2\Delta t^2 k^2)u(\boldsymbol{k},\omega) - u(\boldsymbol{k},\omega)e^{-i\omega\Delta t} \quad (4)$$

Then we obtain

$$vk = \frac{\sqrt{2-2cos(\omega\Delta t)}}{\Delta t} \quad (5)$$

With temporal dispersion errors, the phase of each frequency is $vkt = \frac{\sqrt{2-2cos(\omega\Delta t)}}{\Delta t}t$. Without temporal dispersion errors, the phase of each frequency is ωt. Thus, for the 2nd-order time FD scheme, the phase error from temporal dispersion in the modeled data is

$$\Delta\varphi(\omega,t) = \left[\omega - \frac{\sqrt{2-2cos(\omega\Delta t)}}{\Delta t}\right]t \quad (6)$$

It can be noted that the temporal dispersion is independent of the medium velocity and spatial sampling. It only depends on the time step size and the frequency and propagation time. Hence, the time dispersion error is fully predictable and can be removed or added with filtering. From the equation 6, it can be noted that as a result of temporal dispersion, the phase ωt is mapped into the phase $\omega_d t$ where $\omega_d = \frac{\sqrt{2-2cos(\omega\Delta t)}}{\Delta t}$. Therefore, to predict the temporal dispersion, one way is to map the phase from ωt to $\omega_d t$. This phase mapping can be done using modified inverse Fourier transform, as shown in Wang & Xu (2015)

$$u(t) = \frac{1}{2\pi}\int u(\omega)\, e^{-i\omega_d t} d\omega$$

This modified Fourier transform cannot be implemented with a standard Fast Fourier Transform (FFT) library. However, this phase mapping is equivalent to spectral mapping .i.e., map the spectrum $u(\omega_d)$into spectrum $u(\omega)$. After spectral mapping, the temporal dispersion can be predicted or corrected by using a standard FFT.

The basic procedure for predicting or correcting 2nd-order temporal dispersion is:

1. Convert $\mathrm{u(x,t)}$ into $\mathrm{u(x,\omega)}$ using Fast Fourier Transform (FFT).
2. Apply the time shifting with $\mathrm{u(x,\omega)e^{i\omega t_0}}$ where $\mathrm{t_0}$ is the delay time in the recorded shot gather
3. Map the spectrum $\mathrm{u(\omega_d)}$ into spectrum $\mathrm{u(\omega)}$, i.e., $\mathrm{u(\omega) = u(\omega_d)}$. For predicting temporal dispersion of 2nd-order time FD scheme
$$\mathrm{\omega_d = sgn(\omega)\frac{\sqrt{2-2cos(\omega\Delta t)}}{\Delta t}}.$$
For correcting numerical temporal dispersion of 2nd-order time FD scheme
$$\mathrm{\omega_d = sgn(\omega)\frac{1}{\Delta t}\cos^{-1}\left(1-\frac{\omega^2\Delta t^2}{2}\right)}$$
4. Apply time shifting $\mathrm{u(x,\omega)e^{-i\omega t_0}}$ where $\mathrm{t_0}$ is the delay time in the recorded shot gather
5. Transform $\mathrm{u(x,\omega)}$ into $\mathrm{u(x,t)}$ using Inverse Fast Fourier Transform (IFFT)

For forward modeling, the temporal dispersion correction can be applied as a trace-by-trace post-processing step after simulation. To remove time dispersion artifacts in RTM imaging, the predicted temporal dispersion is added into each input trace before backward propagation. The temporal dispersion in source wavefield and receiver wavefield cancel out while applying the cross-correlation imaging condition. Therefore, we can improve the RTM image at negligible cost.

Synthetic forward modeling example

The earth model consists of simple horizontal density-contrast reflectors and a constant velocity of 3000m/s. A broadband Ormsby source wavelet with peak amplitude at 1s and maximum frequency of 40hz is injected without any delay time. This means that the recorded shot gather will have delay time of 1s. The synthetic shot gather is simulated using time-domain finite difference algorithm. The finite difference algorithm is second order in time and 12th order in space. The spatial grid size is dx=15m and dz=15m.

The left of Fig. 1 shows the gather obtained using large time step of Δt=2ms. It shows typical strong temporal dispersion artifacts on what should be a series of zero phase Ormsby wavelets. The temporal dispersion from 2nd-order time differencing causes the high frequency components to arrive earlier. The early arrivals have little dispersion but the later arrivals have increasing and severe dispersion. The right of Fig. 1 shows the corresponding shot gather after applying the temporal dispersion correction. It can be seen that this method works well to remove the time dispersion errors in numerical modeling. Fig. 2 compares the waveform and amplitude spectrum of reflected waves before and after temporal dispersion correction. It can be seen that the amplitude spectrum of dispersive reflection and dispersion-corrected reflection differs only at high frequency.

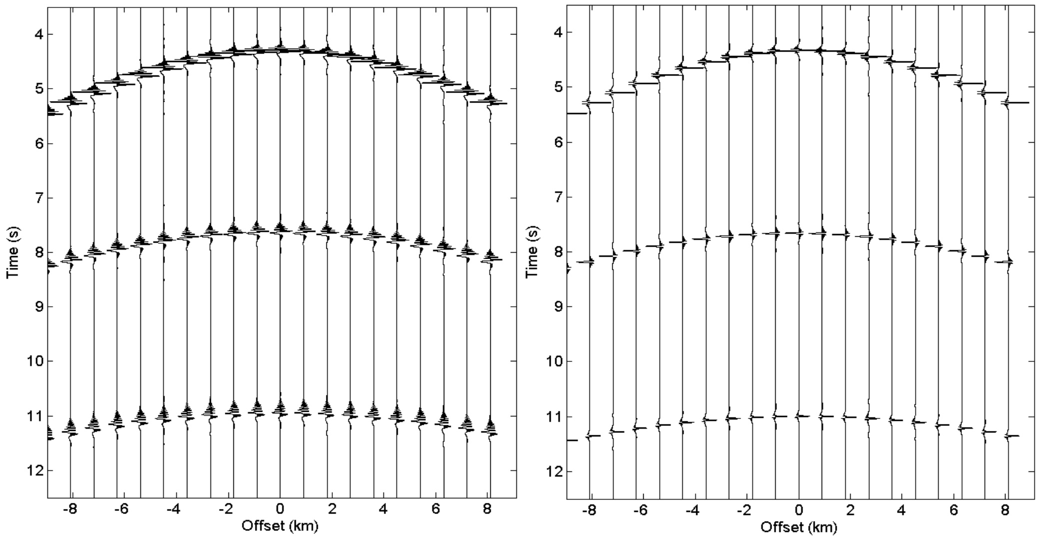

Figure 1 *Left: The synthetic gather generated by using large time step of 2ms. Right: after temporal dispersion correction.*

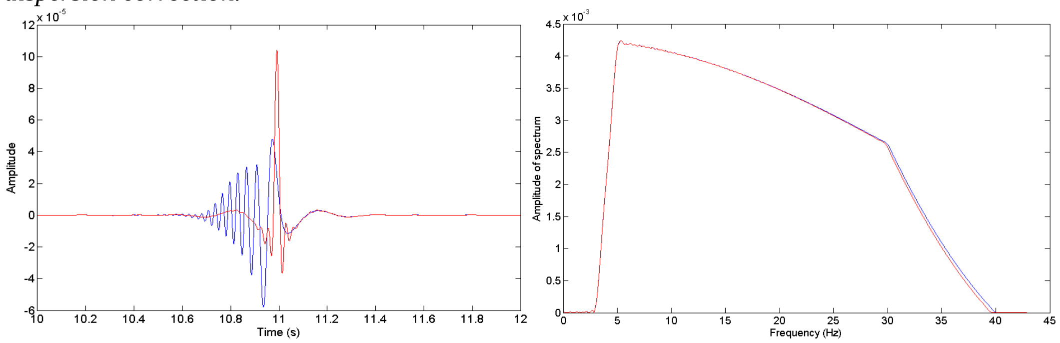

Figure 2 *Left: Comparison of the 3rd reflection at zero-offset before (blue) and after (red) temporal dispersion correction. Right: Comparison of the corresponding amplitude spectrum.*

Real RTM example

We test our method on a complex 2D real marine model with high-velocity salt body. The acoustic equation is used for forward and backward propagations. The Ormsby source wavelet is injected with a delay time of 1s. The maximum frequency used for migration is 40 Hz.

The 2nd-order time FD scheme is used in this example. The predicted temporal dispersion for a time step of 1ms is added into the shot gather. The conventional RTM image obtained by using a large time step of 1ms and original shot gather is shown in the left of Figure 3. The middle of Figure 3 shows the corresponding shot image obtained by using a large time step of 1ms and dispersion-added shot gather. For a detailed comparison, the right panel of Figure 3 compares the image at distance of 7km. It shows that the temporal dispersion has a significant effect on the phase of RTM image and the amplitude of the back-scattering low-frequency noise, which is used as the gradient of tomographic FWI.

The reference RTM image obtained by using very small time step of 0.1ms is shown in the left panel of Figure 4. The right panel compares the waveform of an image trace between the reference and temporal-dispersion corrected RTM images. The perfect match demonstrates that our method works well even for a complex earth model.

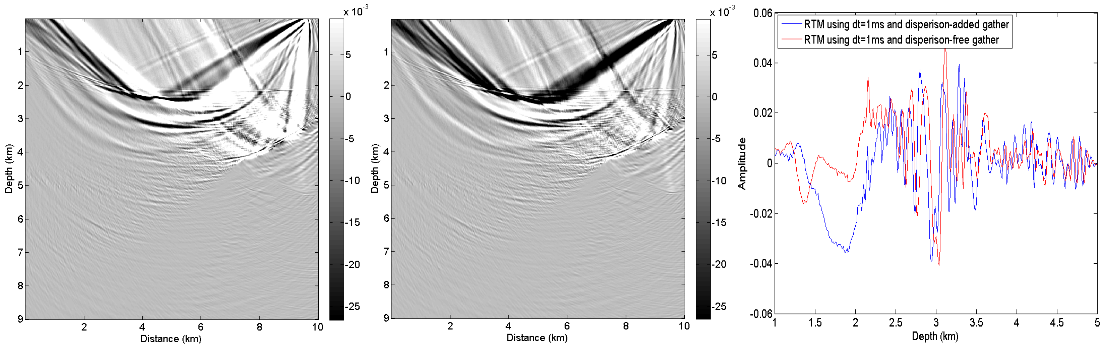

Figure 3 RTM image obtained by using large time step of 1ms and dispersion-free gather (left) and temporal-dispersion-added gather (middle). Right: Comparison of image trace at distance of 7km.

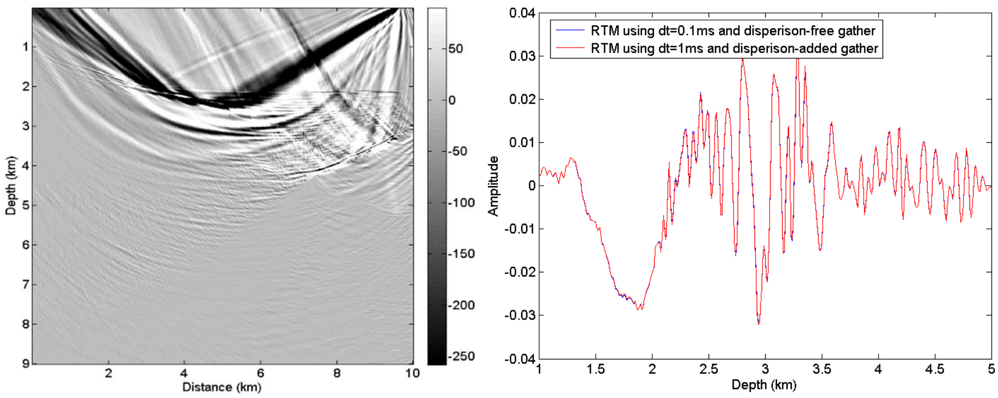

Figure 4. left: Reference RTM image obtained by using small time step of 0.1ms and dispersion-free shot gather. right: comparison of image trace at distance of 7km between the reference RTM image and temporal-dispersion-corrected RTM image (middle of Fig. 3)

Conclusions

In this paper, we show that for both forward modeling and RTM, the temporal dispersion associated with 2nd-order finite difference approximations of time derivative can be predicted and corrected efficiently and accurately using a spectral mapping operation. As a result, a relatively large time step is allowed for wave propagation. Although we demonstrate our method with examples using isotropic models, this method can be applied to a TTI model as well.

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