Direction-vector-based angle gathers from anisotropic elastic RTM

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Summary

We present a method for calculating angle gathers from anisotropic elastic RTM. Reflection angle and azimuth are determined from direction vectors calculated directly from the propagating wavefields. We show that, depending on how the direction vectors are calculated, it may be necessary to convert from group angle to phase angle as part of the angle gather binning process. We give a simple expression for performing this conversion in the case of qP-wave components. Finally, we show that our method is capable of accurate reproduction of theoretical PP reflection coefficients for anisotropic elastic models.

Introduction

Reverse Time Migration (RTM) is now a well-established and preferred tool for pre-stack imaging of complex geological media. Basic isotropic acoustic RTM has been extended to account for various additional physical complexities, e.g. anisotropy (Zhang & Zhang, 2009; Fletcher *et al*., 2009; Duveneck & Bakker, 2011) and elastic media (Sun *et al*., 2006).

Another important area where RTM has been extended in recent years is in the generation of angle-domain common image gathers. Various methods for generating such angle gathers have been proposed and Vyas *et al*. (2011) present a very brief overview. Two common methods for generating RTM angle gathers are (i) extended imaging conditions, and (ii) image binning according to estimated direction vectors for the source, and optionally receiver, wavefield.

In the 1st method (Sava & Fomel, 2006), the usual zero-lag cross-correlation RTM imaging condition is replaced by an "extended" imaging condition in which non-zero lags between source and receiver wavefields are considered. In the most general case of non-zero lags in all 3 space dimensions and in time this leads to a 7D image volume, which may be transformed into angle gathers subsequent to imaging.

The 2nd method (Yoon *et al*., 2011; Dickens & Winbow, 2011) decomposes the normal zero-lag RTM image into angle/azimuth bins according to the estimated local propagation direction of the source and receiver wavefields. Alternatively, as discussed below, the source direction may be used in conjunction with the local structural dip. We note that the term "Poynting vector" is used in both of the above references, even though (as we will see later) the resulting vectors point in different directions for anisotropic media. We will simply use the more generic term "direction vector" to cover both cases.

Zhang & Sun (2009) discuss true-amplitude aspects of angle gather generation from acoustic RTM. Various authors have considered angle gather extraction from isotropic elastic RTM (Yan & Sava, 2008; Yan & Xie, 2011; Zhang & McMechan, 2011). Zhang & McMechan (2011a) present a method for angle gather extraction from 2D VTI elastic RTM, based on an excitation imaging condition with reflection angles being determined from the qP-wave polarization direction. In this work we consider the use of direction vectors for angle gather binning in anisotropic elastic RTM using the conventional cross-correlation imaging condition. Particular attention is paid to the correct binning of gathers according to phase angle.

Direction vector calculation

For acoustic RTM, or pseudo-acoustic RTM in the anisotropic case, we can calculate a direction vector from the gradient of the wavefield as (Yoon *et al*., 2011)

$$\boldsymbol{S}^{\boldsymbol{G}} = -\frac{\partial P}{\partial t}\,\boldsymbol{\nabla} P$$

where P is the (pseudo-)acoustic wavefield amplitude. The superscript on the LHS here denotes that the direction vector is calculated from wavefield gradients.

In the elastic case we can also calculate a direction vector from gradients of wavefield components as above. For example, we could decompose the wavefield into separate (q)P- and (q)S-components and calculate direction vectors from gradients of these. However, it is much simpler to calculate direction vector components directly from the stress tensor and particle velocity components as (Dickens & Winbow, 2011)

$$S_j^E = \sum_k -\tau_{jk}\, v_k$$

where $\boldsymbol{\tau}$, $\boldsymbol{v}$ are the stress tensor and particle velocity vector respectively. The superscript on the LHS here indicates that the direction vector is calculated directly from the elastic fields.

A key point of this work is that, in the anisotropic case, vectors calculated from gradients generally do not point in the same direction as those calculated directly from elastic fields. This point is illustrated in Figure 1, which shows a wavefield propagated through a homogeneous elastic TTI medium. The dip is 45° and Thomsen parameters are ε=0.25, δ=-0.05. The wavefield shown is simply the sum of the normal stress components. The group angle at various points around the outer wavefront is depicted by green arrows, while red and blue arrows show the direction vectors calculated from wavefield gradients ($\boldsymbol{S^G}$) and directly from elastic fields ($\boldsymbol{S^E}$) respectively. It can be seen immediately that $\boldsymbol{S^G}$ points in the direction of the phase angle, whereas $\boldsymbol{S^E}$ points along the group angle. Along the symmetry axis and within the symmetry plane $\boldsymbol{S^G}$ and $\boldsymbol{S^E}$ point in the same direction, since phase and group angle coincide in these cases.

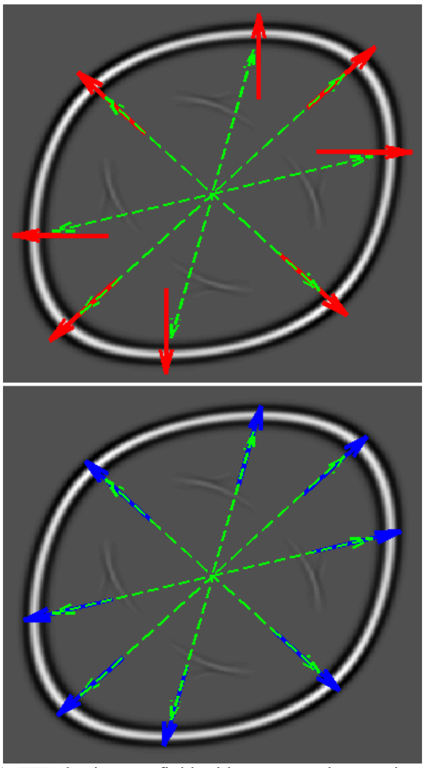

Figure 1: TTI elastic wavefield with group angle at various points around the outer wavefront shown in green, and (top) gradient-based direction vectors ($\boldsymbol{S^G}$) in red; (bottom) elastic-field-based vectors ($\boldsymbol{S^E}$) in blue.

Correct calculation of angle gathers requires that they be binned according to phase angle. Therefore, if we use direction vectors calculated directly from the elastic fields in determining the reflection angle, then we must convert from group to phase angle as part of the final binning of our gathers. This conversion is considered in the following section. Such conversion will not generally be necessary for pseudo-acoustic RTM since direction vectors will usually be calculated from wavefield gradients in this case.

Conversion from group angle to phase angle

Calculation of phase angle (ψ_P) from group angle (ψ_G) is a non-trivial matter and we therefore make some approximations. With the assumption of weak anisotropy, for the qP-wave component we have (Thomsen, 1986)

$$\tan\psi_G = \tan\psi_P \; [1 + 2\delta + 4(\varepsilon - \delta)\sin^2\psi_P]$$

Figure 2 plots the exact qP-wave group angle for a simple homogeneous model, as well as the approximate group angle calculated as above, both as a function of the phase angle. Reasonable anisotropy parameters ($\varepsilon = 0.15, \delta = 0.05$) are used.

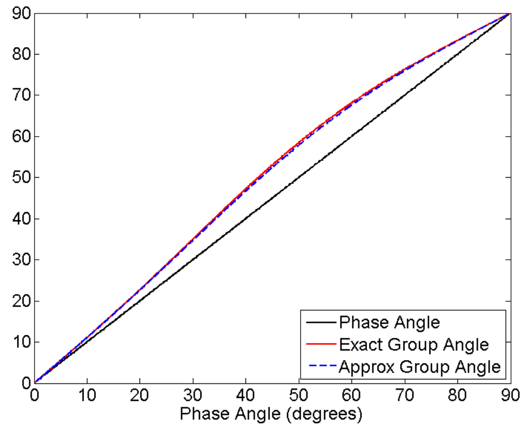

Figure 2: Comparison of qP-wave phase angle with exact and approximate group angle (ε=0.15, δ=0.05)

Note the deviation of the exact group angle from the phase angle. For example, at a phase angle of 40°, the group angle is approximately 47.3°. As the anisotropy increases in strength, this deviation will become more pronounced. Thus it is important to bin angle gathers according to their proper phase angle, particularly if we are using reflection angle bins spanning only 1° or 2°. The second point to note from Figure 2 is that the approximate expression for group angle given above does indeed provide a good estimate in this case. Again, this estimate will become less accurate as the anisotropy increases in strength.

Unfortunately we need to invert the approximate expression for group angle in order to calculate the phase angle. Rearranging gives

$$\tan\psi_P = \frac{\tan\psi_G}{1 + 2\delta + 4(\varepsilon - \delta)\sin^2\psi_P}$$

Our empirical investigations have shown that a reasonable estimate for the phase angle can be found by substituting $\psi_P \approx 0.9\psi_G$ in the denominator, which is in line with the observation that phase angles tend to be slightly smaller than group angles. This substitution leads to our final approximation for the qP-wave phase angle

$$\psi_P \approx \tan^{-1}\left[\frac{\tan\psi_G}{1+2\delta+4(\varepsilon-\delta)\,\sin^2(0.9\,\psi_G)}\right]$$

The effectiveness of this approximate inversion expression is shown in Figure 3, where we compare exact and inverted phase angles for the same anisotropy parameters as considered in Figure 2.

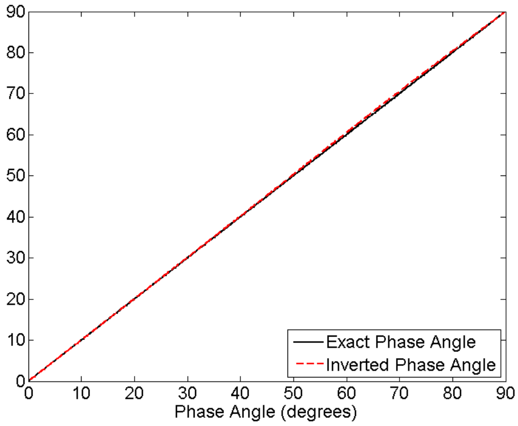

Figure 3: Comparison of original and inverted qP-wave phase angles (ε=0.15, δ=0.05)

Angle gather binning

The reflection angle α may be calculated from direction vectors determined for both the source and receiver field from the expression

$$\cos 2\alpha = \hat{\boldsymbol{S}}_{src} \cdot \hat{\boldsymbol{S}}_{rec}$$

where $\boldsymbol{S}_{src}$ and $\boldsymbol{S}_{rec}$ are the source and receiver field direction vectors respectively. Hats on these vectors denote unit vectors. It is understood here that, if the direction vectors have been found directly from the elastic fields, then the appropriate group-phase angle conversion must be applied. This conversion may either be applied before or after angle binning.

Alternatively, in view of the fact that direction vector estimates for the receiver wavefield tend to be rather noisy, α may be calculated from the source wavefield direction vector and an estimate of local model dips (e.g. Vyas *et al.*, 2011a; Yoon *et al.*, 2011), i.e.

$$\cos\alpha = \hat{\boldsymbol{S}}_{src} \cdot \hat{\boldsymbol{n}}$$

where $\hat{\boldsymbol{n}}$ is the unit vector normal to the local structural dip.

In 3D models a reflection azimuth also needs to be calculated. The definition used here follows that of Vyas *et al.* (2011a). We therefore define the reflection azimuth as the angle between the x-axis and the line forming the intersection of the reflection plane with the horizontal. The reflection azimuth (β) is thus calculated from

$$\cos\beta = \hat{\boldsymbol{m}} \cdot \hat{\boldsymbol{\imath}}$$

where $\hat{\boldsymbol{\imath}}$ is the unit vector in the x-direction and $\hat{\boldsymbol{m}}$ is defined as the unit vector in the direction

$$\boldsymbol{m} = \left(\hat{\boldsymbol{S}}_{rec} \times \hat{\boldsymbol{S}}_{src}\right) \times \hat{\boldsymbol{k}}$$

where $\hat{\boldsymbol{k}}$ is the unit vector in the z-direction. As with the reflection angle, we can also dispense with the need for the receiver-side direction vectors by simply substituting $\hat{\boldsymbol{n}}$ for $\hat{\boldsymbol{S}}_{rec}$.

Prior to calculating reflection angle and azimuth it is usually necessary to apply a small degree of smoothing to the direction vectors in order to eliminate oscillations which could lead to unstable angle estimates. Dickens & Winbow (2011) use spatial smoothing. Yoon *et al.* (2011) present a method for temporal smoothing. While spatial smoothing is generally simpler to apply, temporal smoothing better preserves the spatial locality inherent in the direction vector approach.

Example angle gather results

Here we compare RTM angle gathers with theoretical PP reflection coefficients. The 2D model considered consists of 3 horizontal layers: a water layer, beneath which are 2 VTI layers separated by weak contrasts in material parameters, as summarized in Table 1.

	Layer 1	**Layer 2**	**Layer 3**
P-wave velocity (m/s)	1500	2020	2000
S-wave velocity (m/s)	0	1000	1020
Density (kg/m^3)	1000	2000	2040
$\boldsymbol{\varepsilon}$	0	0.25	0.25
$\boldsymbol{\delta}$	0	0.05	0.05

Table 1: Summary of material parameters used for angle gather calculation.

Direction-vector angle gathers from anisotropic elastic RTM

Angle gathers are calculated using an anisotropic elastic RTM implementation, with angle binning according to direction vectors calculated directly from the elastic fields, i.e. using $\boldsymbol{S}^{\boldsymbol{E}}$ in our previous terminology. All shots and receivers are located at the top of the water layer.

We first consider the isotropic case (where ε=δ=0 in all 3 layers). Other parameters are as given in Table 1. Figure 4 shows the calculated RTM angle gather for the interface between layers 2 and 3 compared against the theoretical PP reflection coefficient. The agreement is clearly excellent. Note in particular that the location of the zero crossing is very accurately predicted by the RTM.

Next we consider the full VTI model as summarized in Table 1. Figure 5 shows the RTM angle gather for the interface between layers 2 and 3, where the angle binning is performed directly according to direction vectors $\boldsymbol{S}^{\boldsymbol{E}}$, i.e. according to group angle. Apart from at small reflection angles, the agreement with the theoretical reflectivity is poor. The location of the zero crossing is overestimated by approximately 6^{o} and the amplitude estimate becomes increasingly poor with larger angles. Figure 6 shows the RTM angle gather for the interface between layers 2 and 3 of the full VTI model, where the angle binning is performed correctly according to phase angle. The relevant phase angles are calculated from direction vectors $\boldsymbol{S}^{\boldsymbol{E}}$, followed by group-phase conversion as detailed earlier. As in the isotropic case, we once again have excellent agreement with the theoretical reflection coefficient, with the location of the zero crossing being accurately predicted.

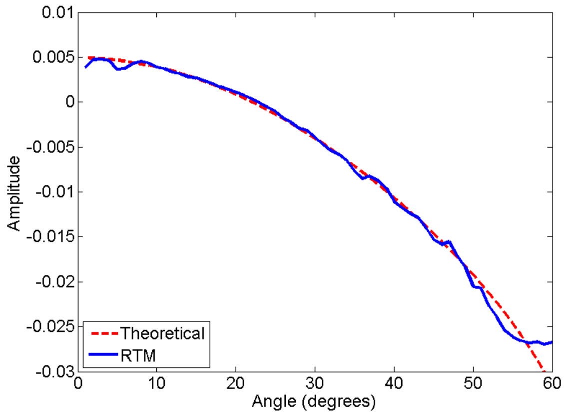

Figure 4: RTM angle gather vs theoretical reflection coefficient. Isotropic model.

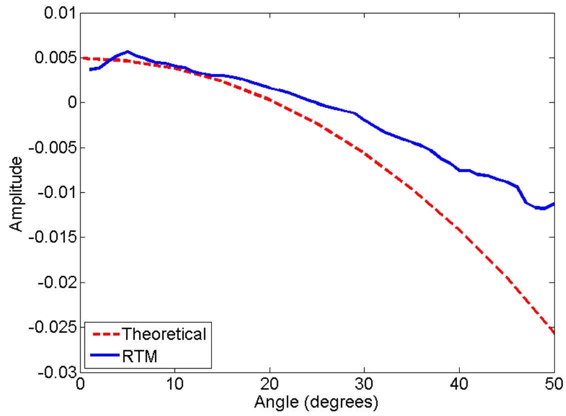

Figure 5: RTM angle gather vs theoretical reflection coefficient. VTI model, binning by group angle.

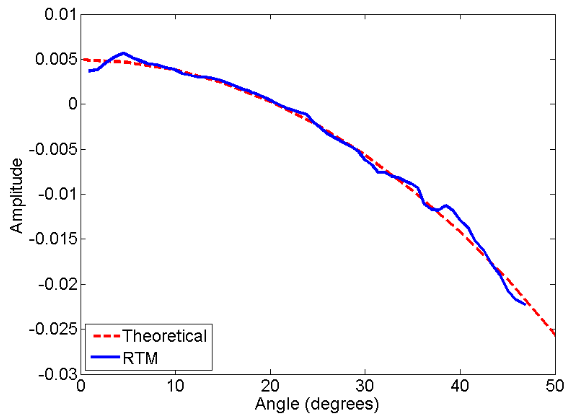

Figure 6: RTM angle gather vs theoretical reflection coefficient. VTI model, binning by phase angle.

Conclusions

We have presented a method for calculating angle gathers from anisotropic elastic RTM, where reflection angle and azimuth are determined from direction vectors calculated either directly from the propagating wavefields or from their gradients. We have shown that, if direction vectors are calculated directly from the elastic fields, it is necessary to convert from group angle to phase angle as part of the angle gather binning process. We derived a simple expression for performing this conversion in the case of qP-waves. Finally, we presented an example which shows that our method is capable of accurate reproduction of theoretical PP reflection coefficients for anisotropic elastic models. In future work we will concentrate on angle binning of converted wave RTM images.

Acknowledgements

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